

MESOSCOPIC TRANSPORT AS MANY-BODY PHYSICS*

Frederick Green

Centre for Quantum Computer Technology
School of Physics, The University of New South Wales
Kensington NSW 2052, Australia

and

Department of Theoretical Physics
Research School of Physical Sciences and Engineering
The Australian National University
Canberra ACT 0200, Australia

Mukunda P. Das

Department of Theoretical Physics
Research School of Physical Sciences and Engineering
The Australian National University
Canberra ACT 0200, Australia

1. INTRODUCTION

Mesoscopic transport physics has grown astoundingly in the last decade. This kind of physics continues to astonish by the delicacy of its architectures, and of the experiments done on them. In every way, mesoscopics provides an ideal test-bed for many-body theory, since the reduced dimensionalities and small scales naturally amplify the effects of inter-electron correlations. However, there is a deeper reason. It rests with the nature of electronic transport and its microscopic origin in electron-hole fluctuations.

The sustained development of this field depends, therefore, on a matching capacity to understand dynamic electron processes at length scales reaching down to the atomic. Aside from its intrinsic scientific value, such an understanding is crucial to the commercial design and integration of novel electronic devices.

A standard bulk-averaging analysis will break down when actual device sizes approach the typical mean free paths for scattering, or the natural screening length.

* To be published in *Proceedings of the 25th International Workshop on Condensed Matter Theories (CMT25), Canberra 2001*.

In such systems, much more care has to be taken. In the last two decades a body of work, widely seen as having its origins in Landauer's early insights [1], has enjoyed great success in mesoscopic transport. The best known result within this approach is the Landauer formula for the quantized conductance of a one-dimensional wire.

To many practitioners of mesoscopics, the Landauer formula seems to possess quasi-magical powers in that it claims to explain a staggering diversity of transport measurements (as witness the authoritative list in Reference [2]). It is a formula that, in its self-confessed simplicity, bases itself strictly on *one-body* physics and nothing else. As a result, it contains none of the important electron correlations that come into prominence for low-dimensional mesoscopic structures. More serious is the fact that Landauer's model ignores a series of indispensable physical bounds on the behavior of the electron gas (see Sections 2 and 3 below for details).

A further important point is that many would-be proofs of Landauer's popular formula insist upon the paradoxical notion that two (or even more) chemical potentials, each thermodynamically distinct, should coexist within a single, closed, electrical circuit when it is out of electro-chemical equilibrium. On that account also, Landauer's prescription falls well short of internal consistency. We will deal with this issue on the way to setting out a proper, microscopic view of small-scale transport.

For a good general reference on mesoscopics, see the book by Ferry and Goodnick [1]. On the status of mesoscopic noise theory as popularly received, refer to Blanter and Büttiker [2]. We also mention the recent critiques of Landauer-Büttiker-Imry theory in the review of mesoscopic transport by Agraït *et al.* [3].

In the evolution of this new branch of condensed-matter physics, concepts of small-scale current behavior cannot be divorced from their roots within microscopic electron correlations. These roots are firmly grounded in the theory of charged Fermi liquids [4–6]. Within that long-established canon, we have two aims:

- to help motivate mesoscopics as an inherently *many-body* discipline, and
- to encourage a reciprocal interest in the fresh opportunities for testing many-body ideas, as offered by mesoscopic physics.

The relationship between microscopic many-body physics and mesoscopic device physics is indeed two-way. Not only does the former hold the foundations for a true understanding of the latter; mesoscopics, in turn, manifests (often quite graphically) the action of the conservation laws. It is the microscopic conservation laws that give many-body theory its universal reach and its great explanatory power [7].

Attention falls on a primary conservation law: gauge invariance. On it, one can build a firm and trustworthy picture of dynamic fluctuations in an already classic mesoscopic system: the one-dimensional metallic wire, or quantum point contact (QPC). This case study highlights the need for *any* mesoscopic transport model to be, first and genuinely, a many-electron theory. It also points to the prospect of researching strong correlations through nonequilibrium current noise, a tool that so far has been unfamiliar to many-body physics.

In Section 2 we review the general conditions applying to an open and externally driven system of mobile electrons, metallic ones in particular. Our discussion recalls a series of insightful, if less often discussed, microscopic analyses. These have been freely available in the literature over the last decade [8–13]. After that, in Sec. 3 we

analyze the compressibility of an open conductor. This parameter, closely allied to the electron-hole fluctuations, is an exact invariant of the system. It is absolutely unaltered by nonequilibrium transport. This result extends beyond the free electron gas, to systems that are subject to strongly inhomogeneous Coulomb screening. Nonuniform electronic conductors are encountered everywhere in mesoscopic physics, and internal screening leads to the substantial suppression of their compressibility. This effect too is fully inherited by a system when it is driven out of equilibrium. Last, we foreshadow the effect of exchange-correlation renormalization on the compressibility.

Charge conservation governs the compressibility of a charged Fermi liquid. It also governs its response right through the high-field nonlinear regime. Section 4 retraces our kinetic noise theory with reference to the critical QPC noise experiment of Reznikov *et al.* [14], whose features raise some outstanding issues of principle. These are resolved in a natural way within our kinetic many-body perspective, and compressibility is the key to their resolution. We conclude in Sec. 5.

2. TRANSPORT IN OPEN ENVIRONMENTS

A fundamental problem in mesoscopic transport is the explicit consideration of what it means, in microscopic terms, to address “open environments” that no longer seem to admit direct treatment within a closed Hamiltonian formalism. Over the last decade, a number of important comprehensive papers has appeared. Taken as a whole, they contribute substantially towards the realization of a logical and calculable microscopic approach to transport at mesoscopic scales. This is so not just in linear response, but also in truly nonequilibrium situations. The authors responsible for this body of work are:

Sols [8];

Fenton [9,10];

Magnus and Schoenmaker [11,12]; and

Kamenev and Kohn [13].

We first examine the fundamental principles that they articulate, dissect, and in large measure resolve. It hardly needs saying that their critiques of mesoscopic theory stand upon a time-tested and rigorous analytic tradition, from Maxwell and Boltzmann through Fermi and Landau.

Neither is it a surprise to find that such first-principles descriptions of the mesoscopic electron gas begin – and end – with charge conservation. However, its practical outworking can be subtle. For instance, unlike an ideally closed system of carriers not subject to exchange with the outside, the global charge neutrality of a real – open – conductor is not automatic, so that a nontrivial demonstration is required. Nonetheless, the physics of charge and number conservation applies as strictly as ever.

a. Gauge Invariance

Sols [8] first established that an open metallic conductor is globally gauge invariant if, and only if, the microscopic equation of motion for the electrons *explicitly*

includes the contribution from each source and sink of current in the problem. These are located at the device's interfaces with the outlying reservoirs. The reservoirs themselves remain in (local) equilibrium.

Sols' formalism introduces a critical separation of physical roles within the mesoscopic description. Thus, the dynamical action of charge injection and removal (which brings about the system response) is entirely separate from the equilibrating action of the reservoir leads. The macroscopic leads act, in the first instance, not as regulators of the current but to pin and stabilize the nonequilibrium state of the intervening device. *The current is not conditioned by the lead equilibria.*

The clear implication of Sols' examination is that the current sources/sinks operate dynamically in their own right. They have nothing to do with the locally unchanging state of the reservoirs. According to Ref. [8], injection and extraction must act, and be represented, *manifestly*.

Transport descriptions that take no notice of the entry and exit of flux are necessarily faulty. Ref. [8] shows that the sources and sinks are irreducible elements in a gauge-invariant model. Thus it is theoretically not possible to cut them out of open-system transport merely at convenience, as if these essential flux exchanges could be equated to a remote (and forgettable) background effect [15].

From Sols, a major point follows:

- *A microscopically conserving model of transport and fluctuations must manifestly account for all influx and efflux of current at the open boundaries.*

Let us now focus on the quiescent reservoirs. They lie outside the conducting sample, in which the interesting transport phenomena all occur. At the same time, they are in intimate electrical contact with it. As we have remarked, Ref. [8] shows that the reservoirs' microscopic function is distinct from the supply and recovery of the (nonequilibrium) current fluxes.

Therefore the reservoir leads have two dominant properties, which are *not* dynamic but, rather, thermodynamic. The macroscopic leads are

- (i) always neutral, and
- (ii) always in local equilibrium.

In other words, they pin the carriers asymptotically (via the local Fermi levels in the respective leads, as well as the thermodynamic bath temperature) to their permanent and locally unchanging equilibrium state. The corollary is that

- *The nonequilibrium carrier states must connect seamlessly to each invariant equilibrium state that locally and uniquely characterizes each of the leads.*

b. Screening

Shortly after Sols' formal proof, *Fenton* [9,10] made a detailed quantum-kinetic analysis of mesoscopic transport. He highlighted the dominance of charge neutrality of the reservoirs. Through Thomas-Fermi screening, the asymptotic carriers *confine* the electromotive force (EMF) so that it is spatially coextensive with the driven conductor, across whose boundaries it appears.

Fenton drew a deep conclusion from his first-principles quantum analysis. The quantized conductance observed in one-dimensional (1D) wires is explainable if *and*

only if its underlying model includes the screening response of the reservoirs at the sample interfaces. This is, of course, a consequence of Gauss' theorem applied to the electron gas (the perfect-screening sum rule [4], also recalled by Sols [8]). It entails the statement that

- *The interface regions of an open mesoscopic sample – the actual sites of the carriers' physical transition to the asymptotic charge-neutral state – must be included as part of the theoretical description of the transport.*

As in every electron-gas problem, screening is an integral part of the procedure here. Unless it operates, Landauer's quantized mesoscopic conductance can find no reconnection with microscopic quantum kinetics [9]. Fenton's analysis requires that – rather than the common practice of demoting metallic-electron screening to some *ad hoc* perturbative afterthought [15] – a mesoscopic transport description make such collective screening its centerpiece.

c. Hamiltonian Description

So far we have restated the prime roles of gauge invariance, global neutrality, and equilibrium in charged open systems. This is sufficient to set up a kinetic theory in the mesoscopic regime, not only in the weak-field limit but well away from equilibrium [16,17].

Since energy dissipation is a real and central effect in such cases, it is easy to conclude that a canonical Hamiltonian approach is not feasible. To the extent that a (dissipative) dynamical equation cannot be generated from a standard hermitian Hamiltonian form, one might think that the kinetic description of real carrier dynamics, with its energy losses, is destined to remain formally incomplete.

This incompleteness is not inevitable. A central fact that should be taken into account is the multiply connected topology of a closed circuit. An elegant microscopic formalism for nonequilibrium, energy-dissipating and closed mesoscopic circuits has been proposed by *Magnus and Schoenmaker* [11,12]. They point out that the toroidal topology of the circuit (this includes the battery) admits valid solutions to Maxwell's equations which, though locally conservative, are *globally nonconservative* along complete paths around the hole of the torus.

Magnus and Schoenmaker construct the gauge-invariant Hamiltonian for a closed and *multiply connected* system of driven carriers. The classic minimal-coupling expression for the electronic Hamiltonian is faithfully maintained. What is novel is that the physical solution, resulting from the geometrical reconnection, will dissipate energy from the battery along any full circuit path.

In this way, real processes can be described without altering the locally conservative Hamiltonian structure of the interacting electrons. All of the powerful quantum kinetics of Kadanoff-Baym [18], or of Keldysh [19], becomes immediately applicable in principle.

Under very general conditions, the authors formally prove that the external power P supplied by the source is dissipated according to the standard formula $P = IV$, in terms of circuit current I and EMF V . The result is intuitively clear, but challenging to prove. The derivation is fully quantal, and valid at any device scale.

The work of Magnus and Schoenmaker dovetails with that of Sols and Fenton.

The treatments are all equivalent in the end; the latter two stress the governing role of the boundary conditions within a charge-conserving description of a finite mesoscopic device that is *electrically open to the outer leads*. On the other hand, the former approach restates the critical fact that any circuit path containing the device must be *closed*, and that the topology of the closure makes possible (i) long-range neutrality, (ii) dissipation, and hence (iii) dynamical stability within a unified Hamiltonian description.

d. Uniqueness of the Chemical Potential

Finally, the recent many-body derivation of Landauer’s conductance formula by *Kamenev and Kohn* [13] is noteworthy.

- (a) It sets the Landauer result within a solidly orthodox quantum-kinetic formulation, free of the unnecessary phenomenology that has weakened previous such proofs;
- (b) it provides a detailed and self-contained treatment of collective screening;
- (c) by applying Kubo response theory in its appropriate form, Ref. [13] naturally preserves the logical structure of the microscopic fluctuation-dissipation theorem;
- (d) it demonstrates the unphysical character of a purely heuristic scheme now widely accepted in mesoscopics: namely, the practice of “driving” all currents by a kind of pseudodiffusive process, generated by a virtual mismatch in the chemical potential across various leads.

At the two-body level, a consequence of Refs. [8-12] is to rule out that last artifice in all its variations, for they all lead inevitably to the violation of gauge invariance [20].

It is Kamenev and Kohn’s work [13], however, that drives home the fictional nature of pseudodiffusive phenomenology, by straightforward *ab initio* evaluation of Kubo’s conductance formula for a 1D wire. Just one parameter is needed: the equilibrium chemical potential. Thus

- *the microscopic properties of a mesoscopic system are uniquely determined by one chemical potential, and one only: that of the reference equilibrium state.*

e. Microscopics and Phenomenology

Each of the approaches enumerated above counters a viewpoint commonly advanced by other models [2,15,21,22]. As already mentioned, the electrical currents in such phenomenologies are generated and (allegedly) sustained by passive connection of the mesoscopic device to two or more large reservoirs at unequal potentials. Instead of being allowed their correct thermodynamic values, measured from the bottom of their occupied *local* conduction bands, the chemical potentials in the reservoirs are taken from a global zero of energy which itself has no unique physical significance. This immediately breaks the canonical linkage that ties together the *physical* carrier density and the *physical* compressibility of the collective system [20]. We return to the compressibility below.

In the dominant conception of mesoscopics, carriers are never driven dynamically through the conductor, from one lead to another, by the real field. Rather, they fall passively, and by sheer happenstance, over the associated Fermi-level drop. All

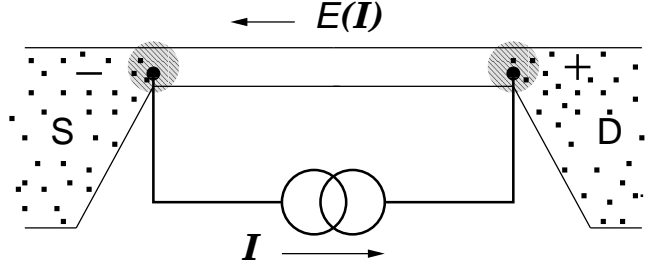


Figure 1. Mesoscopic transport in a nutshell. A closed, i.e. toroidal, loop includes the conductor under test (clear central area). The loop is the essential supplier of the current I , either by a generator as shown or, equivalently, by the EMF from a battery. The source (S) and drain (D) reservoirs ensure (a) *inelastic dissipation* of the electrical power dumped to the circuit, (b) *screening* of all fields including the driving field $E(I)$ across the nonequilibrium region, and (c) *continuity* of the carrier distribution. These dynamically competing processes are located at the interfaces (shaded areas). The boundary conditions illustrated here are necessary and sufficient for a microscopically conserving description. (This Figure is taken from our Ref. [23]).

that can be known of the dynamics are the asymptotic equilibrium states, and the asymptotic probability for a single carrier to drop over the potential cliff.

This picture is of no help in explaining the *energy dissipation* that has to accompany open-system transport. Pseudodiffusive phenomenologies have no access to anything but generic equilibrium properties. From a detailed kinetic perspective, those models cannot contribute to understanding the critical nonequilibrium processes unfolding within the conductor, and at its interfaces with the reservoirs. These processes dissipate energy. Dissipation is out of scope.

Figure 1 summarizes the physical structure of a mesoscopic system open to its macroscopic electrical surroundings, within which all measurements take place. The closed loop supplies the flux, as specified in Ref. [11] while, in accordance with Ref. [8], the flux entry and exit points maintain global conservation as perceived by a test charge *within the wire*. The large reservoirs maintain thermodynamic stability by pinning the local Fermi level at the interfaces, by providing energy dissipation (Ref. [11]), and by confining the electromotive forces via screening (Ref. [9]).

By simple logic, the phenomenological scenarios cannot allow the reservoirs themselves to reconnect electrically “at infinity” and thus to close the circuit. If they did so, the resulting coexistence of several *different* chemical potentials (required for pseudodiffusive “transport”) would pose an immediate consistency problem [11,13]. There is no flux generator, and thus no replenishment of the ever-growing charge imbalance between the reservoirs. Therefore an adiabatically sustainable, current-carrying steady state cannot be described within such an approach.

By contrast, in a standard many-body description all the dynamic and dissipative effects that determine both transport and noise conform to the microscopic conditions articulated in Refs. [8–13], and summarized in Fig. 1. Without those normative boundary conditions (which include the batteries and/or current generators), a well-controlled model cannot be set up. That is the message of the works we have recalled.

3. A CANONICAL SUM RULE: COMPRESSIBILITY

As is well known, the physics of the electron gas is strongly conditioned by a set of canonical theorems, the sum rules [4]. Several of the sum rules are directly linked to number conservation, and thus to gauge invariance. They, if nothing else, should be identically satisfied by any model of transport and fluctuations.

The electron-gas sum rules share a common template. On the left-hand side, a physical quantity is given as a function of the thermodynamic and *single-particle* parameters (temperature; chemical potential; density etc.). This is equated, on the right-hand side, to a *two-particle correlation function* [24].

Conceptually, the sum rules are closely related to the fluctuation-dissipation theorem [25,26]; once again we see a one-body parameter, the electronic conductance, on one side of the relation. On the other side, it is uniquely determined by an irreducible two-body response function: the current-current correlator. Equivalences such as these express the microscopic conservation of energy, momentum, and particle number at each elementary electron-hole vertex in the system's quantum many-body expansion [5].

With this background, let us examine the content and mesoscopic consequences of the simplest of the sum rules, that for compressibility. On its own, that rule is a surprisingly powerful constraint on the behavior of mesoscopic conductors. For the full and formal details, see our Refs. [16,17,20].

a. The Open Electron Gas

Consider a driven, metallic conductor of volume Ω bounded by *equilibrated* and *charge-neutral* reservoirs. Here we invoke the boundary conditions discussed above; see Sec. 2.a. That the well-defined active region, including the interfaces, must be fixed and finite follows from Fenton's microscopic analysis; see Sec. 2.b.

Within the device, let N be the number of mobile carriers, and let $\Delta N \sim \langle N^2 - \langle N \rangle^2 \rangle$ be the mean-square fluctuation of N , where the averaging will be made precise shortly. Since global neutrality holds,

- N is unconditionally invariant for all transport of charge through the system.

The total number N can be accounted for in two ways. We can integrate the time-dependent nonequilibrium distribution $f_\alpha(t)$ for the configuration-space label $\alpha = (\mathbf{r}, \mathbf{k})$ (spin and many-valley indices are implied). Alternatively, we can add up all the electrons in Ω for the equilibrium case, where the one-body distribution is

$$f_\alpha^{\text{eq}} = \frac{1}{1 + \exp[(\varepsilon_k + U(\mathbf{r}) - \mu)/k_B T]}.$$

The (quasiparticle) kinetic energy is ε_k ; the Hartree mean-field potential is $U(\mathbf{r})$, and is generally a functional of the density distribution. The global chemical potential μ is fixed by the outer reservoirs.

Very simply,

$$\sum_{\alpha} f_{\alpha}(t) = N = \sum_{\alpha} f_{\alpha}^{\text{eq}}. \quad (1)$$

The origin and meaning of this equation are clear. We add up all contributions to N over the space of wavevectors \mathbf{k} and over all \mathbf{r} within the real-space volume Ω . The operation \sum_{α} denotes this configurational integration. Since the region of physical interest is fixed and unconditionally neutral, N is always exactly balanced by the unchanging positive background. Equation (1) follows.

We now sketch the corresponding fluctuation relation. It is a consequence of Gauss' law, expressed through Eq. (1). The *difference function*

$$g_{\alpha}(t) = f_{\alpha}(t) - f_{\alpha}^{\text{eq}}$$

measures the local out-of-balance component of the ensemble. Eq. (1) is equivalent to

$$\sum_{\alpha} g_{\alpha}(t) = 0. \quad (2)$$

If we take variations of the difference function, for example with respect to the chemical potential proper to the reservoir leads (this is done by changing the leads' asymptotic electron density *and that of its neutralizing positive background*), then

$$\sum_{\alpha} \frac{\partial g_{\alpha}(t)}{\partial \mu} \equiv \sum_{\alpha} \left(\sum_{\alpha'} \frac{\delta g_{\alpha}(t)}{\delta f_{\alpha'}^{\text{eq}}} \frac{\partial f_{\alpha'}^{\text{eq}}}{\partial \mu} \right) = 0. \quad (3)$$

We have used the variational chain rule to partly unpack the rich inner structure of this fluctuation; the functional derivative $\delta g_{\alpha}(t)/\delta f_{\alpha'}^{\text{eq}}$ is a specialized Green function whose form is calculable from the kinetic equation for $f(t)$ [17]. It is the centerpiece of our approach.

b. Free-Electron Compressibility

Equation (3) is the cardinal element in deriving the nonequilibrium compressibility sum rule. First, the equilibrium mean-square fluctuation in local carrier number is given by

$$\Delta f_{\alpha}^{\text{eq}} \equiv k_{\text{B}} T \frac{\partial f_{\alpha}^{\text{eq}}}{\partial \mu} = f_{\alpha}^{\text{eq}}(1 - f_{\alpha}^{\text{eq}}), \quad (4a)$$

which lets us compute the total mean-square fluctuation in N :

$$\Delta N \equiv k_{\text{B}} T \frac{\partial N}{\partial \mu} = \sum_{\alpha} \Delta f_{\alpha}^{\text{eq}}. \quad (4b)$$

Second, the exact *nonequilibrium* mean-square fluctuation is [17]

$$\Delta f_{\alpha}(t) = \Delta f_{\alpha}^{\text{eq}} + \sum_{\alpha'} \frac{\delta g_{\alpha}(t)}{\delta f_{\alpha'}^{\text{eq}}} \Delta f_{\alpha'}^{\text{eq}}. \quad (5)$$

From Eq. (3) we get, at last,

$$\sum_{\alpha} \Delta f_{\alpha}(t) = \Delta N = \sum_{\alpha} \Delta f_{\alpha}^{\text{eq}}. \quad (6)$$

The compressibility for an undisturbed free-electron gas is standard [4]:

$$\kappa \equiv \frac{\Omega}{N^2} \frac{\partial N}{\partial \mu} = \frac{\Omega}{N k_{\text{B}} T} \frac{\Delta N}{N}. \quad (7)$$

Note that, so far, we have taken our variations (and the associated fluctuation structures) with all internal fields held *fixed*; that is, we freeze the potential $U(\mathbf{r})$. Shortly, we will relax this constraint. At this stage our treatment is precisely analogous to the Lindhard model of the electron gas.

Our result is immediate. Eqs. (1) and (6) are exact within the nonequilibrium kinetic description. Therefore

- *The invariant compressibility of an open mesoscopic conductor, driven at any current, is given by Eq. (7).*

A simple feature of Eq. (7) is seen in its Maxwellian limit. In that case, $\Delta f^{\text{eq}} = f^{\text{eq}} \propto e^{\mu/k_{\text{B}}T}$. The ratio $\Delta N/N$ is unity. That renders the right-hand expression equal to the inverse of the pressure $P = N k_{\text{B}} T / \Omega$. Eq. (7) then states the classical, ideal-gas result

$$\kappa_{\text{cl}} = P^{-1}.$$

When the system is degenerate, then $\Delta f^{\text{eq}} < f^{\text{eq}}$ and

$$\kappa_{\text{free}} = \kappa_{\text{cl}} \frac{\Delta N}{N} < \kappa_{\text{cl}} \quad (8)$$

(we denote the free “Lindhard” compressibility by κ_{free} in anticipation of the next subsection). This shows the stiffening action of degeneracy on the system, in loose analogy with van der Waals’ concept of the excluded hard-core volume. The effect clearly persists, *with no change at all*, out of equilibrium. This is despite the fact that the underlying microscopic form of the dynamic $\Delta f(t)$ is far more complex than its generic equilibrium limit, Δf^{eq} . The power of the compressibility sum rule comes precisely from its total, and surprising, insensitivity to external driving forces.

We have just exhibited a general result for nonequilibrium electronic conductors. It is the direct consequence of conservation, acting in the context of the boundary conditions of Sec. 2. These boundary conditions are the only ones fully consistent with microscopic principles that hold, universally, in a many-body situation. The mesoscopic electron gas is one such.

c. Screened Compressibility

It is time to focus on the reaction of the internal fields to the carrier fluctuations. Intuitively, any such fluctuation costs additional electrostatic energy. The system will minimize the excess internal energy by self-consistent screening of the initial charge fluctuation. This is Coulomb-induced suppression, of which the Coulomb blockade is one of the more dramatic instances.

Let us start with the self-consistent equilibrium response:

$$k_B T \frac{\delta f_\alpha^{\text{eq}}}{\delta \mu} = \frac{\delta}{\delta \mu} (\mu - U(\mathbf{r})) k_B T \frac{\partial f_\alpha^{\text{eq}}}{\partial \mu} = \left(1 - \frac{dU}{dn} \frac{\delta n(\mathbf{r})}{\delta \mu} \right) \Delta f_\alpha^{\text{eq}}. \quad (9a)$$

We have made the usual Ansatz for $U(\mathbf{r})$; one assumes that it depends directly on the local density so that $U \equiv U[n(\mathbf{r})]$. This holds – even mesoscopically – for typical length scales in excess of the inverse Fermi wavenumber k_F^{-1} . With slightly different physics, it applies exactly to quantum-well-confined systems such as the two-dimensional electron gas formed within heterostructure materials (the basis of very many mesoscopic devices) [27].

The local-density Ansatz provides a constitutive relation to close Eq. (9a). We need another. Denoting by $\tilde{\Delta} f_\alpha^{\text{eq}}$ the left-hand (mean-square) fluctuation in the equation, we have the integral identity

$$\frac{\delta n(\mathbf{r})}{\delta \mu} = \frac{1}{k_B T} \langle \tilde{\Delta} f_\alpha^{\text{eq}} \rangle_{\mathbf{k}},$$

where we locally take the trace of $\tilde{\Delta} f_\alpha^{\text{eq}}$ over the space of wavevectors \mathbf{k} . This decouples from the spatial dependence on \mathbf{r} , as is appropriate in a local-density treatment.¹ After minor rearrangement, Eq. (9a) goes to

¹Our basic screening result does not rely on the local-density assumption. In the wider case, Eq. (9a) reads

$$\tilde{\Delta} f_\alpha^{\text{eq}} = \Delta f_\alpha^{\text{eq}} \left(1 - \int_{\Omega} d\mathbf{r}' \frac{\delta U(\mathbf{r})}{\delta n(\mathbf{r}')} \frac{\delta n(\mathbf{r}')}{\delta \mu} \right).$$

Since the mean-field potential couples only to the density, one can still integrate over \mathbf{k} , leaving the real-space (albeit nonlocal) integral relation

$$\int_{\Omega} d\mathbf{r}' \left(\delta(\mathbf{r} - \mathbf{r}') + \frac{\partial n(\mathbf{r})}{\partial \mu} \frac{\delta U(\mathbf{r})}{\delta n(\mathbf{r}')} \right) \frac{\delta n(\mathbf{r}')}{\delta \mu} = \frac{\partial n(\mathbf{r})}{\partial \mu},$$

in which the extended response function $\delta U(\mathbf{r})/\delta n(\mathbf{r}')$ carries the screening. After solving the equation we arrive at $\tilde{\Delta} f_\alpha^{\text{eq}}$, essentially as for Eq. (9b) below:

$$\tilde{\Delta} f_\alpha^{\text{eq}} = \left\{ \frac{\delta n(\mathbf{r})/\delta \mu}{\partial n(\mathbf{r})/\partial \mu} \right\} \Delta f_\alpha^{\text{eq}}.$$

$$\tilde{\Delta}f_{\alpha}^{\text{eq}} = \frac{1}{1 + \frac{\langle \Delta f_{\alpha}^{\text{eq}} \rangle_{\mathbf{k}}}{k_{\text{B}}T} \frac{dU(\mathbf{r})}{dn}} \Delta f_{\alpha}^{\text{eq}} < \Delta f_{\alpha}^{\text{eq}}. \quad (9b)$$

The leading factor is the inverse static dielectric function. (In its general form it can be seen in the final equation of the preceding footnote.) The total mean-square number fluctuation, in the presence of self-consistent screening, is

$$\tilde{\Delta}N = \int_{\Omega} d\mathbf{r} \langle \tilde{\Delta}f_{\alpha}^{\text{eq}} \rangle_{\mathbf{k}}.$$

The screened analog to Eq. (7) can now be stated:

$$\kappa \equiv \frac{\Omega}{N^2} \frac{\delta N}{\delta \mu} = \frac{\Omega}{N k_{\text{B}}T} \frac{\tilde{\Delta}N}{N}. \quad (10)$$

Omitting the relevant proof, already fully detailed in Ref. [17], we state the major outcome for nonequilibrium mesoscopic conductors:

- *In the presence of screening, the invariant compressibility of an open, nonuniform mesoscopic conductor – driven at any current – is given by Eq. (10).*

Equation (10) stands in relation to the random-phase approximation (RPA) [4] as Eq. (7) stands in relation to Lindhard. There are differences, however.

- (a) The conductor must be *inhomogeneous*. Otherwise $U(\mathbf{r}) = 0$ everywhere, and $\kappa = \kappa_{\text{free}}$. This is merely the reflection of a known fact [4]: in a perfectly uniform system, long-range RPA screening is not manifested.
- (b) The conductor must be *degenerate*. Classically, the Maxwell form of f^{eq} entails equipartition of energy. The Coulomb term in the distribution decouples exactly from the kinetic one, leading to the relation $\tilde{\Delta}N = \Delta N$. Again, $\kappa = \kappa_{\text{free}} = \kappa_{\text{cl}}$.

An accessible and striking example of screening-induced suppression occurs in the quasi-two-dimensional electron gas (2DEG) confined within a heterojunction quantum well. In Fig. 2 we display the compressibility of a 2DEG at room temperature, as found in any production-grade AlGaAs/InGaAs/GaAs device. At sheet densities higher than 10^{11} electrons per cm^2 , we see the rapid onset of the Coulomb suppression in κ where, by comparison, degeneracy acts rather more mildly.

The electrons, firmly held within the (engineered) quantum well, exhibit their strong mutual repulsion via the density dependence of their Hartree potential $U[n(\mathbf{r})]$. The consequent reduction of carrier fluctuations by self-consistent feedback is an important element of practical heterojunction-device design [27].

Exactly the same Coulomb reduction of Δf must also reveal itself in the nonequilibrium current *noise*, since the same electron-hole pair processes that underpin the compressibility do so equally for noise [17,20]. We explore this in the next Section. Before doing so, let us add a novel twist to the physics of nonequilibrium compressibility.

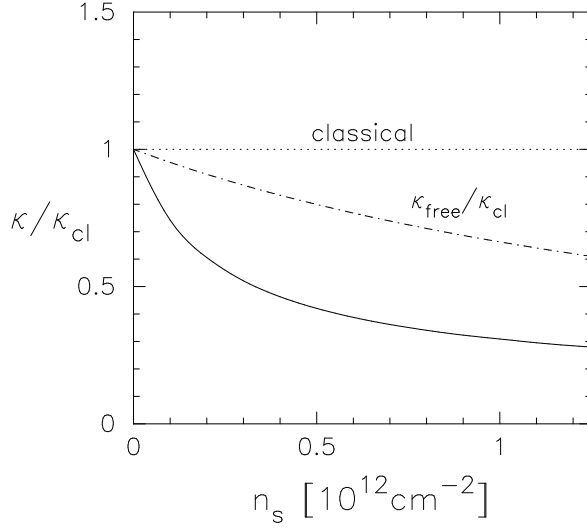


Figure 2. Compressibility of a quantum-confined two-dimensional (2D) electron gas, normalized to that of the 2D classical gas and plotted as a function of sheet electron density. Dashed line: the compressibility of free conduction electrons decreases with increasing density, owing to their degeneracy (see Eq. (8) in the text). Full line: the self-consistent compressibility of interacting 2D electrons (Eq. (10) in text). With increasing density, the mean-field Coulomb energy of a confined carrier population will increase. That makes them collectively more resistant to compression over and above κ_{free} , and provides a prime example of a mesoscopic *many-body* effect. Since electron fluctuations scale with κ , mesoscopic noise must directly manifest these (and other) many-body effects.

d. Exchange and Correlation

So far, we have treated the electron gas regardless of exchange-correlation processes. Already in this Section, we have seen the many-body nature of fluctuation physics come to the fore in the compressibility response of a nonequilibrium conductor. The introduction of exchange-correlation corrections is not a trivial matter; it has many levels of refinement. Here, to give the flavor of what could be achieved for transport, we will discuss these effects in terms of an additional potential $U_{\text{xc}}[n(\mathbf{r})]$.

In density-functional theory, the exchange-correlation potential augments the Hartree term. The total compressibility can be written by inspection from Eq. (9b):

$$\kappa_{\text{tot}} = \frac{\Omega}{N^2 k_B T} \sum_{\alpha} \frac{\Delta f_{\alpha}^{\text{eq}}}{1 + \frac{\langle \Delta f_{\alpha}^{\text{eq}} \rangle_{\mathbf{k}}}{k_B T} \left(\frac{dU}{dn}(\mathbf{r}) + \frac{dU_{\text{xc}}}{dn}(\mathbf{r}) \right)}. \quad (11)$$

At high densities the Hartree, or RPA, term U dominates. In the low-density limit the counterbalancing exchange-correlation term U_{xc} is expected to dominate. An interesting density regime should then exist, in which the nonuniformity of, say, a strongly confined quantum channel competes against the shorter-ranged part of the electron-electron interaction (with exchange). As far as we know, none of these phe-

nomena has been discussed systematically in the specific context of nonequilibrium transport.

We remark that the above approach, via an expression for U_{xc} which may be quite simple (and certainly static), has much in common with the local-field correction long familiar in electron-gas theory [28]. As Eq. (11) shows, our procedure amounts to embellishing the RPA with a term that undoes the RPA’s over-correction of the internal energy of the system; *quasiparticle renormalization* never enters explicitly. Indeed, the seminal quantum formulation of Boltzmann transport by Kadanoff and Baym [18] (which informs many nonequilibrium kinetic theories, including ours) neglects the interaction corrections to single-particle propagation, while keeping particle-particle collision terms that are of comparable order in the coupling strength.

The result of this partial inclusion of exchange-correlation is a fully conserving and calculable description, but one that is conceptually incomplete (more obviously so at low carrier densities). In a mean-field treatment, the Boltzmann–Kadanoff–Baym collision terms will yield something close to the exchange-correlation potential U_{xc} . However, the corresponding self-energy effects are not there.²

It is outside our present brief to look into the complexities of nonequilibrium-transport effects induced by quasiparticle renormalization, and how to include them consistently (let alone their yet-to-be-discussed mesoscopic implications). The inclusion of renormalization within a kinetic equation of Boltzmann type, in a microscopically gauge-invariant way, is a challenging and still unfinished theoretical project. It is the subject of intensive thought by specialists. We can do no better than to cite a small subset of this work. Several excellent summaries, with appropriate references, appear in a volume on many-body kinetics collated by Bonitz [29]. Out of many detailed studies, two that address different facets of these problems are Refs. [30] and [31].

4. NONEQUILIBRIUM FLUCTUATIONS: NOISE

In the previous Section we reviewed the essentially *many-body* fluctuation origin of a central result: the compressibility sum rule for the electron gas. We have discussed how all the quantum correlations that condition this constraint are faithfully retained when a system of (fluctuating) conduction electrons is

² An interim fix for this incompleteness is to set the effective mass of the carriers in the model to that of the renormalized quasiparticles at the Fermi surface. However, that parameter itself depends on the Landau quasiparticle parameter F_1^s which is, by definition, the p -wave coefficient of the two-body exchange-correlation interaction. Again we discern the quite intricate self-consistency between quasiparticle renormalization and collision properties. A beautiful relation, the *forward-scattering sum rule* [see e.g. W. F. Brinkman, P. M. Platzman, and T. M. Rice, Phys. Rev. **174**, 495 (1968)], ties together all of the Landau partial-wave amplitudes, including the determinant of the effective mass F_1^s , and the s -wave term F_0^s which governs the exchange-correlation component of the compressibility [4]. This sum rule is not an expression of conservation, but rather of the fermion antisymmetry that quasiparticles inherit from the underlying “free”-electron modes.

- (1) *open* to large external reservoirs that uniquely fix its equilibrium state, and
- (2) *driven* out of equilibrium by external current sources (or, equally well, by external EMFs) in a way that always preserves global gauge invariance for the open conductor.

The self-consistent structure of the compressibility rule is intimately tied to charge conservation. This internal consistency must therefore be preserved even in an approximate transport description. *A model that violates the compressibility sum rule is an incorrect model.*

Efforts to set up a sum-rule preserving transport theory would be purely academic if the nonequilibrium fluctuations of the electron gas could not be probed effectively in the laboratory. While conductance is readily measured, the information that it carries on electron-hole fluctuations is very coarse-grained. To study the fluctuation structure, one must access the electrical noise generated within the device. If our arguments are correct, the noise leads directly to significant many-body effects that are invisible in the conductance.

One of the first well-controlled measurements of nonequilibrium noise in a 1D mesoscopic channel was the quantum-point-contact experiment of Reznikov *et al.* [14]. Among several phenomenological treatments of this system, the best known is the Landauer-Büttiker model [2]. It predicts a noise-power spectrum S that is dominated by shot noise.

The predicted shot noise has a well developed sequence of peaks and troughs as the channel’s carrier density is swept from the “pinch-off” (zero-density) region, upwards through the region where the Fermi level successively accesses the quantized conduction subbands of the QPC. At the same time, the conductance G increases in a series of more-or-less delineated stepwise mesas. Ideally, the steps in G come in integer units of the Landauer value $e^2/\pi\hbar$ (empirically, they hardly ever do so).

It is normal experimental practice to measure G and S , as functions of carrier density, for fixed values of the source-drain driving voltage. Although the fit to experiment is by no means perfect, notably for noise, the standard noise models [2] give a semiquantitative account of the observations. A major innovation introduced by Reznikov *et al.* was to obtain an additional series of noise data for fixed values of source-drain *current*. Here, a glaring discrepancy immediately stands out, between the data and the standard noise prediction.

In Fig. 3 we show a single trace of S as published by Reznikov *et al.* [14], straddling the first subband. It is evident that the prediction of a strictly monotonic S by the Landauer-Büttiker noise theory totally misses the strong peak structure in the data. The authors of the experiment note this. However, they make no further comment on what is a wholly unexpected – and unexplained – outcome.

There is another, purely kinetic-theoretical explanation for the Reznikov *et al.* peak anomalies. Here we sketch it in the formulation of Thakur and the present writers [32]. Our model, which is strictly conserving, obeys the compressibility sum rule by construction (for further details see Refs. [20] and [23]).

From a standard kinetic Boltzmann equation for the carrier distribution $f(t)$, we generate an associated equation of motion for the dynamic mean-square fluctuation $\Delta f(t)$ [16,17]. The full 1D solution, obtained analytically in the collision-time

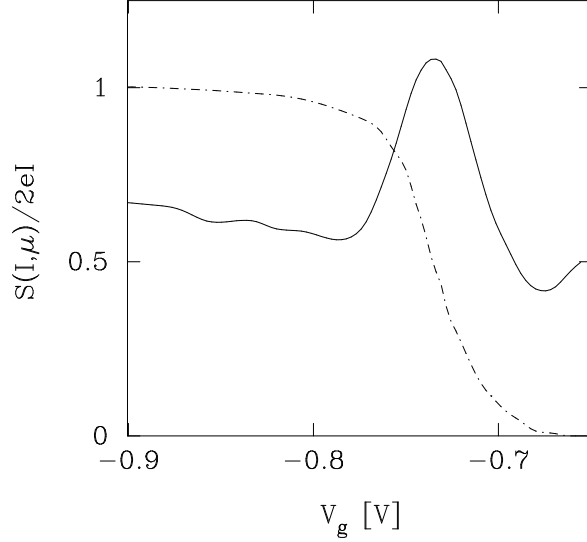


Figure 3. Low-frequency excess noise spectrum S of a quantum point contact as described in Ref. [14], measured at fixed source-drain current $I = 300\text{nA}$ and plotted as a function of carrier density in the first subband. The density is regulated by the gate potential V_g . Full line: S (with threefold enlargement). At the subband threshold, where the Fermi level first accesses the subband, S peaks strongly. Dot-dashed line: the outcome of Landauer-Büttiker noise theory [2] using the measured conductance. This strictly monotonic prediction totally misses the peak. When the channel is depleted, the carriers become classical; there the plateau in S is qualitatively similar to shot noise, but falls short of the classical asymptotic value $2eI$.

approximation, allows us to write the exact nonequilibrium noise.

Normalized to the classical shot-noise expression $2eI$ at source-drain current I , the free-electron excess noise (at low frequency) is

$$\frac{S}{2eI} = \frac{\kappa}{\kappa_{\text{cl}}} \frac{eI}{Gm^*L^2} \left(\tau_{\text{in}}^2 + 2 \frac{\tau_{\text{el}}\tau_{\text{in}}^2}{\tau_{\text{el}} + \tau_{\text{in}}} - \frac{\tau_{\text{el}}^2\tau_{\text{in}}^2}{(\tau_{\text{el}} + \tau_{\text{in}})^2} \right). \quad (12)$$

The set of parameters is as follows. The classical compressibility $\kappa_{\text{cl}} = 1/nk_{\text{B}}T$ is in terms of the subband carrier density n , keeping only the lowest subband of the QPC as in [14]. The device conductance is G , while the effective electron mass is m^* and L is the operational length of the 1D channel. The inelastic and elastic collision times are, respectively, τ_{in} and τ_{el} . Since the sample is *ballistic* – at least in the neighborhood of equilibrium – the mean free path associated with τ_{el} is essentially the ballistic device’s length, L . Phonon emission, however, will cause the inelastic mean free path to become progressively shorter than L as the current intensifies.

Equation (12) has two principal features. First, the *compressibility* enters directly into the nonequilibrium excess spectrum. At high density,

$$\frac{\kappa}{\kappa_{\text{cl}}} \rightarrow \frac{k_{\text{B}}T}{2\varepsilon_{\text{F}}}$$

where ε_{F} is the Fermi energy. Even far from equilibrium, the excess noise necessarily

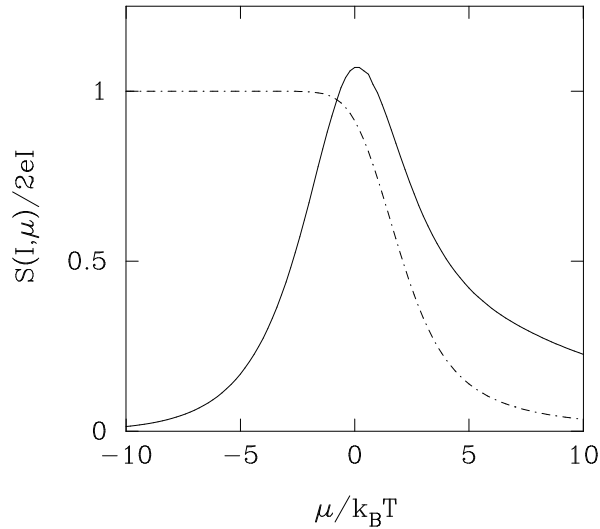


Figure 4. Kinetic-theoretical simulation of the excess noise spectrum of a quantum point contact, corresponding to the conditions of Fig. 3. Density is controlled by the chemical potential μ . Full line: S (with threefold enlargement). At the subband threshold our strictly conserving solution for S shows a strong peak that is *quantitatively close* to the observed structure. Dot-dashed line: the outcome of Landauer-Büttiker noise theory [2], using our kinetically computed conductance. This purely monotonic prediction still misses the peak. Close to depletion of the channel, our simulation does not exhibit a plateau as seen in Fig. 3. We ascribe this to the absence of the physically quite distinct shot-noise contribution in our present calculation, which addresses only the excess *thermal* noise contribution to S .

scales with the ambient temperature. The greater the degeneracy, the smaller S becomes owing to the factor above. At low density, the electrons are classical and

$$\frac{\kappa}{\kappa_{\text{cl}}} \rightarrow 1.$$

In the classical limit the factor is independent of temperature.

Second, the structure of the last right-hand term in Eq. (12) makes S very sensitive to the ratio of the two collision times. If, as expected, τ_{in} decreases substantially with the applied current I , the magnitude of the excess noise will be correspondingly reduced. This expectation provides the basis for our simulation of the result quoted in Fig. 3. It is displayed in Fig. 4.

Figure 4 shows our direct computation of S from Eq. (12), for environmental variables chosen to match the data of Fig. 3. We parametrize the inelastic time as a function of I and n to account for the enhanced rate of phonon release. This is then fed into Eq. (12). The elastic collision time, being set by impurity scattering only, remains unaffected throughout.

We see a strong peak structure in the noise. It arises from the competition among κ , I , and $\tau_{\text{in}}(I, n)$. The corresponding Landauer-Büttiker curve holds no trace of any peak behavior.

It is important to keep in mind that the Landauer-Büttiker approach badly violates the compressibility sum rule, and indeed charge conservation [20]. Our standard kinetic method respects both. Violation of gauge invariance is built right into the Landauer-Büttiker noise formula. That is known even to its architects [2].

5. SUMMARY

In this review we have stressed that the understanding of nonequilibrium mesoscopic transport is indivisible from its microscopic origins in the electron gas. The degenerate electron gas is, of course, a prime example of a *multi-particle* system.

We first discussed the boundary-condition physics of an open mesoscopic conductor. We did this with reference to a series of searching works by a number of authors. They are works that deserve a wider audience – and a far more seriously engaged one – than the present arbiters of mesoscopic fashion seem able, or perhaps willing, to muster. Certainly the papers we have recalled provide, with considerable power, a prelude for the microscopic analysis of open systems which we have applied in our own nonequilibrium investigations.

Within the kinetic paradigm, we discussed how the well-known *compressibility sum rule* for the electron gas is rigidly satisfied, not just at equilibrium but far from it. This result governs the structure of nonequilibrium fluctuations in the driven system. Through the action of the compressibility, the noise spectrum of a mesoscopic wire becomes the distinctive signature of Fermi-liquid correlations in low-dimensional, nonequilibrium metallic-electron transport.

From the compressibility, much more might be learned about low-dimensional current correlations. We illustrated our fundamental many-body approach with the graphic example of high-field noise in a one-dimensional metallic conductor. Its spectrum is remarkable for its sensitivity to many-body effects, and especially for the tight way in which these are orchestrated by microscopic conservation.

It is obvious that any description of mesoscopic noise, especially one that claims to be well formulated, must always conserve charge and particle number. There is little here to soften the implications for other approaches to mesoscopics [2,15,21,22]. The absolutely indispensable action of conservation is plain. So is the direct, and by now documented, evidence of its violation [20]. Moreover the specific predictions of microscopic kinetics, notably for the nonequilibrium noise of a quantum point contact, are a significant element in clarifying the discussion through new experiments.

An expanded program to measure nonequilibrium noise in quantum point contacts would be an excellent place to look for a breakdown of nonconserving models. It would also verify the microscopic alternative, whose conserving properties are necessary but not sufficient for validity. The detailed exploration of electron-electron correlations, through renormalization of the nonequilibrium-noise spectrum, poses an intriguing and (to our knowledge) untouched possibility. Much has to be done to realize that potential.

ACKNOWLEDGMENT

In loving appreciation of Marjorie Ann Osborne, 1955–2002: sister-in-law and manuscript facilitator par excellence. FG.

REFERENCES

- [1] D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, Cambridge, 1997).
- [2] Ya. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1 (2000).
- [3] N. Agraït, A. Levy Yeyati, and J. M. van Ruitenbeek, *Preprint* cond-mat/0208239.
- [4] D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966).
- [5] P. Nozières, *Theory of Interacting Fermi Systems* (Benjamin, New York, 1963).
- [6] A.A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).
- [7] Recall, for example, the Ward-Pitaevski-Takahashi identities which tie the renormalization of single-particle dynamics to the two-particle correlation vertex. The identities are the outworking of microscopic conservation [5]; they also entail the *sum rules* [4,5] that must be satisfied by the electron-hole fluctuations, and thus inevitably the current noise, of an electron gas. See also Ref. [20] below.
- [8] F. Sols, *Phys. Rev. Lett.* **67**, 2874 (1991).
- [9] E. W. Fenton, *Phys. Rev. B* **46**, 3754 (1992).
- [10] E. W. Fenton, *Superlattices and Microstruct.* **16**, 87 (1994).
- [11] W. Magnus and W. Schoenmaker, *J. Math. Phys.* **39**, 6715 (1998).
- [12] W. Magnus and W. Schoenmaker, *Phys. Rev. B* **61**, 10883 (2000).
- [13] A. Kamenev and W. Kohn, *Phys. Rev. B* **63**, 155304 (2001).
- [14] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, *Phys. Rev. Lett.* **75** 3340 (1995).
- [15] Y. Imry and R. Landauer, *Rev. Mod. Phys.* **71**, S306 (1999).
- [16] F. Green and M. P. Das, *J. Phys.: Condens. Matter* **12**, 5233 (2000).
- [17] F. Green and M. P. Das, *J. Phys.: Condens. Matter* **12**, 5251 (2000).
- [18] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, Reading, 1962).
- [19] L. V. Keldysh, *Zh. Exp. Teor. Fiz.* **47**, 1515 (1964) (*Sov. Phys. JETP* **20**, 1018 (1965)).
- [20] F. Green and M. P. Das, in *Noise and Fluctuations Control in Electronic Devices*, edited by A. A. Balandin (American Scientific Publishers, New York, 2002), Ch. 3.
- [21] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 1997).
- [22] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University

- Press, Cambridge, 1997).
- [23] F. Green and M. P. Das, *Fluctuation and Noise Letters* **1**, C21 (2001).
 - [24] From this generic structure, we see that no description of transport – mesoscopic or otherwise – can avoid the need to address truly multi-particle correlations. Not all self-styled mesoscopic theories meet this requirement [2,15,21,22].
 - [25] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry*, revised edition (North-Holland, Amsterdam, 2001).
 - [26] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II: Nonequilibrium Statistical Mechanics*, second edition (Springer, Berlin, 1991).
 - [27] C. Weisbuch and B. Vinter, *Quantum Semiconductor Structures: Fundamentals and Applications*, (Academic Press, San Diego, 1991).
 - [28] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990).
 - [29] M. Bonitz (editor), *Progress in Nonequilibrium Green's Functions* (World Scientific, Singapore, 2000).
 - [30] K. Morawetz, P. Lipavský, and V. Špička, *Prog. Part. Nucl. Phys.* **42**, 147 (1999).
 - [31] Yu. B. Ivanov, J. Knoll, and D. N. Voskresensky, *Nucl. Phys. A* **672**, 313 (2001).
 - [32] J. S. Thakur, M. P. Das, and F. Green, in process.